Inadequacy of an Inverted Pendulum Model of Human Gait

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Introduction

There remains some disagreement on the source of anterior ground reaction forces (*Fx*) in normal gait. For some, a passive "roll-off" results from the position of the body's center-of-mass (COM) forward of its base of support [1], and to others, an active "push-off" results from plantarflexion power generation at the ankle [2]. The former theory essentially models the body as an inverted pendulum and has support in walking toys developed as far back as the late 1800s, and in recent passive dynamic robots [3]. A limitation in these devices is the need for a downward slope upon which to "walk." That we can walk on level and upwardly sloped surfaces suggests that we do indeed input mechanical energy. Based upon these observations, and our experience with pathological gait, we hypothesized that COM position alone would be insufficient to fully explain *Fx*.

Statement of Clinical Significance

By exposing limitations of a gravity-based, pendulum model for gait, we provide additional, though indirect, support for interpretation of joint powers in clinical gait analyses.

Methodology

We simulated an inverted pendulum (Fig. 1) with three initial conditions corresponding to gait initiation (Sim1), mid-stance (Sim2), and initial contact (Sim3). In each case, we used three anthropometric profiles that spanned our pediatric patient population: pendulum lengths r equaled the height of the full-body COM in standing (0.714, 0.891, 1.04 m) and were paired with three pendulum masses m (23, 41, 69 kg), chosen to approximate a 50th percentile 7 yr old boy, 12 yr old girl, and 18 yr old boy, respectively. The third profile matched data for an adult in our normal database, allowing comparison with actual *Fx* values. Using the methods of Pai & Patton [4], we derived Eqs. 1 and 2 for the anterior and vertical ground reaction forces at the pivot (*Fx* and *Fy*, respectively, where θ, ω, and α are angular position, velocity, and acceleration). Simulations numerically integrated planar equations of motion. As a final test of this model, we defined an inverted pendulum from the center-of-pressure (COP) to the calculated COM during an actual gait trial (six-camera Vicon 370 system, three AMTI force plates, and a 13-segment six degree-of-freedom full-body model in Visual3D), and used the kinematics of this pendulum in Eqs. 1 and 2, comparing results to data obtained from the force plates.

$$Fx = m\left[\left(\ddot{r} - r\omega^2\right)\cos\theta - \left(r\alpha + 2\dot{r}\omega\right)\sin\theta\right] \tag{1}$$

$$Fy = m\left[\left(\ddot{r} - r\omega^2\right)\sin\theta + \left(r\alpha + 2\dot{r}\omega\right)\cos\theta\right] + mg \tag{2}$$

Results

For all simulations, *Fx* gradually increased through a range of values reasonably close to those for normal gait (peaking near 25% bodyweight), but did so over an extended time period. When data for the third profile were compared to actual data from a matched normal subject, peak *Fx* was delayed 1.08 s for Sim1, 0.07 s for Sim2, and 0.30 s for Sim3 (Fig. 2). When the full-body model was used to define the COP-COM inverted pendulum, there was little agreement during single support between predicted and actual *Fx*, but reasonable agreement for *Fy* (Fig. 3).

Discussion

We were initially impressed by the agreement between simulated and actual *Fx* curves for Sim1, but the marked delay in reaching the peak force made this explanation for gait untenable. Because it could be argued that we unduly penalized the model by using a zero initial velocity, Sim2 provided an initial anterior velocity equivalent to normal gait. This was the "easiest" simulation for the model, and it performed best here, but important energy losses at initial foot contact were ignored [5]. We artificially overcame these losses in Sim3 by again providing an initial anterior velocity equivalent to normal gait; yet despite this, kinetic energy losses were still obvious in an abnormal 50% reduction in anterior velocity at mid-stance (not shown). The COP-COM inverted pendulum seems to model *Fy* reasonably well (Fig. 3), but its inability to predict *Fx* with any accuracy suggests that it under represents normal efforts to modulate shear forces. This is consistent with literature identifying ankle push-off power at terminal stance as the most efficient strategy to sustain gait [5]. We conclude that although normal gait may take advantage of gravitational effects, an inverted pendulum model alone lacks sufficient mechanical energy to sustain gait.

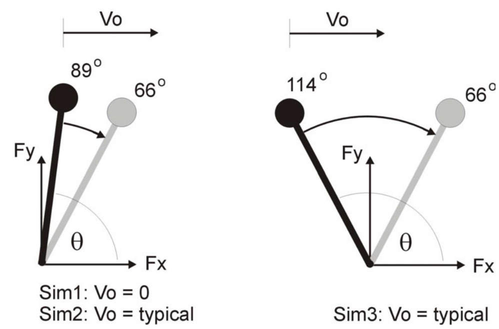

Fig. 1. Inverted pendulum simulations. Initial anterior velocities Vo were either zero, or typical for a child of the age being simulated.

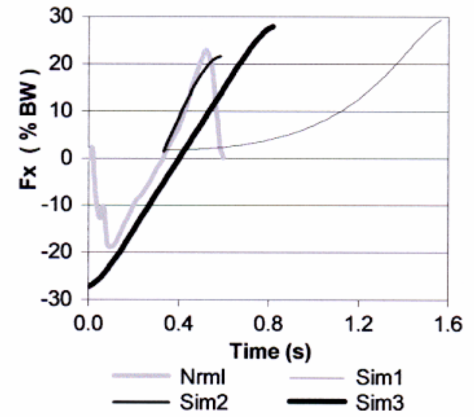

Fig. 2. Normal versus simulated anterior forces for the 3rd anthropometry profile.

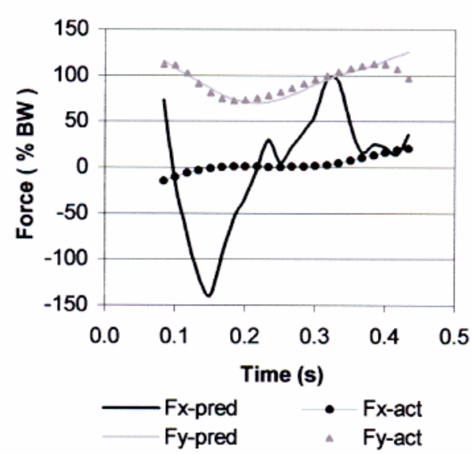

Fig. 3. Predicted versus actual forces during single support. Anterior (Fx) and vertical (Fy) forces are shown.

References

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